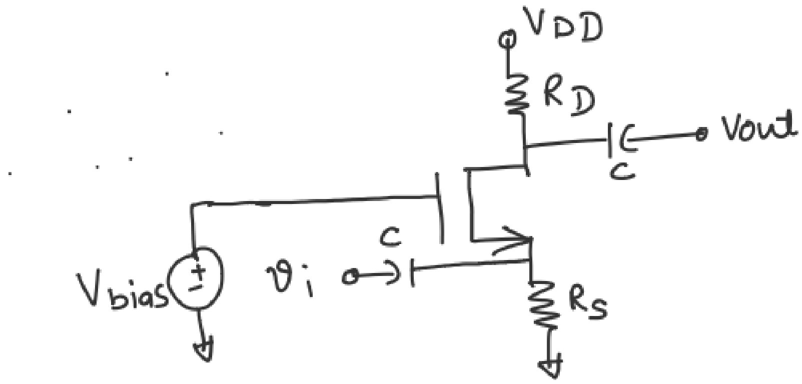
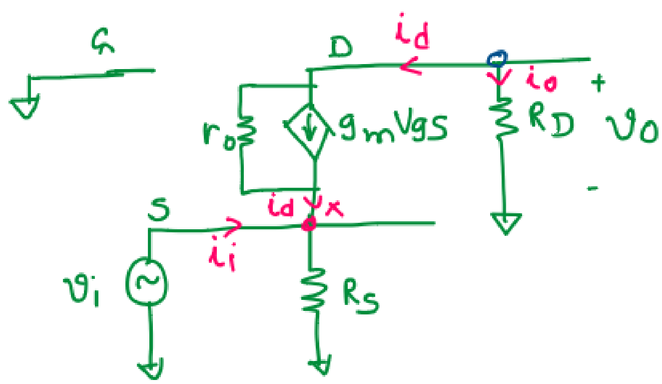


For small signal Analysis, equivalent circuit



Voltage gain and input impedance

Applying KCL at node x,

$$i_i - \frac{v_i}{R_S} + i_d = 0 \quad \ldots (1)$$

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$$i_d = g_m V_{gs} + \frac{v_o - v_i}{r_o} \quad -(2)$$

Applying KCL at output node

$$i_o + i_d = 0$$

$$\Rightarrow i_d = -\frac{V_o}{R_D} \quad \ldots (3) \qquad \left(\because i_o = \frac{v_o}{R_D}\right)$$

Equating eq^n (2) & (3)

$$g_m v_{gs} + \frac{v_o - v_i}{r_o} = -\frac{v_o}{R_D}$$

Also, $v_{gs} = -v_i$

$$\Rightarrow g_m v_i + \frac{v_i}{r_o} = \frac{v_o}{R_D} + \frac{v_o}{r_o}$$

$$\Rightarrow v_i \left[g_m + \frac{1}{r_o} \right] = v_o \left[\frac{1}{R_D} + \frac{1}{r_o} \right]$$

$$\Rightarrow \boxed{\frac{v_o}{v_i} = A_v = \frac{g_m + \frac{1}{r_o}}{\frac{1}{R_D} + \frac{1}{r_o}}} \quad \text{.... (4)}$$

If $r_o = \infty$, then $A_v = g_m R_D$

Now, equating eq^n (1) & (3)

$$i_i - \frac{v_i}{R_S} = \frac{v_o}{R_D}$$

$$\Rightarrow i_i - \frac{v_i}{R_S} = \frac{A_v \times v_i}{R_D}$$

$$\Rightarrow i_i - \frac{v_i}{R_S} = \frac{v_i}{R_D}\left[\frac{g_m + \frac{1}{r_o}}{\frac{1}{R_D} + \frac{1}{r_o}}\right]$$

$$\Rightarrow i_i - \frac{v_i}{R_S} = v_i\left[\frac{g_m + \frac{1}{r_o}}{1 + \frac{R_D}{r_o}}\right]$$

$$\Rightarrow i_i = v_i \left[\frac{g_m + \frac{1}{r_o}}{1 + \frac{R_D}{r_o}} + \frac{1}{R_S} \right]$$

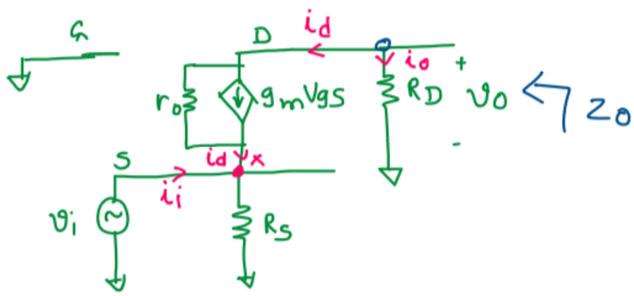
$$\Rightarrow \frac{i_i}{v_i} = \frac{g_m + \frac{1}{r_o}}{1 + \frac{R_D}{r_o}} + \frac{1}{R_S}$$

$$\Rightarrow \frac{1}{z_i} = \frac{g_m r_o + 1}{r_o + R_D} + \frac{1}{R_S}$$

$$\Rightarrow z_i = \frac{r_o + R_D}{1 + g_m r_o} \, || \, R_S$$

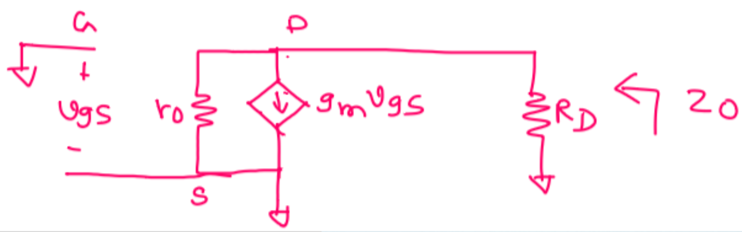
If $r_o = \infty \Rightarrow z_i = 1/g_m \, || \, R_S$

Output Impedance:



For output impedance, let's consider all the independent sources as zero.

⇒ considering $v_{in}=0$, R_S will get short circuited.



since, $v_{gs} = 0$, dependent current source will also be zero

And $Z_o = r_o \| R_D$

That means for CG amplifier,

$$Z_o = r_o \| R_D$$

$$Z_i = R_S \| \left[\frac{r_o + R_D}{1 + g_m r_o} \right]$$

$$A_v = \left[\frac{g_m + \frac{1}{r_o}}{\frac{1}{R_D} + \frac{1}{r_o}} \right]$$