Single Sideband Suppressed Carrier

Friday, 30 April 2021 5:15 PM

SSB:

Its bandwidth efficient modulation scheme.

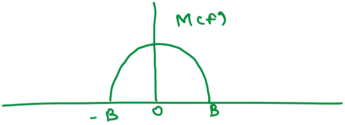
→ Both in DSBSC and AM, two sidebands are transmitted.

- upper sideband
- Lower side band

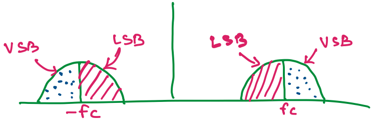
→ Each contains the complete information of the baseband signal m(t).

→ For baseband signal m(t) with bandwidth B, DSBSC and AM requires 2B bandwidth.

→ single side band (SSB) modulation requires B Hz of bandwidth.

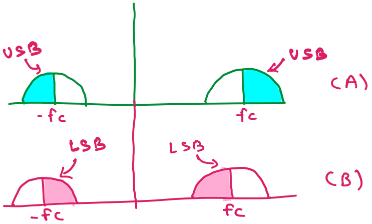


DSB-SC spectrum



→ In SSB, since LSB and USB contains same information, any one of them is suppressed before transmission.

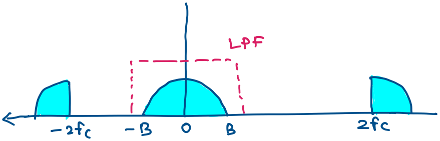
SSB-SC spectrum



* At the receiver, SSB signal can be coherently demodulated.

If $\phi(t)$ is SSB signal, then multiplying it with $\cos(\omega_c t)$ shifts the spectrum right and left by ω_c.

→ If the SSB spectrum shown in fig. A is multiplied with $\cos(\omega_c t)$ then the resultant spectrum is shown below.



→ By passing that signal through a low pass filter, the desired Base band signal can be retrieved.

* Mathematical representation of SSB signal:

Hilbert Transform:

→ With the help of Hilbert Transform, we can represent the SSB signal.

If $x(t)$ is some signal, then $x_h(t)$ or $H\{x(t)\}$ is the Hilbert transform of $x(t)$.

$$x_h(t) = H\{x(t)\} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\alpha)}{t-\alpha}\, d\alpha$$

$$\Downarrow$$

$$x(t) * \frac{1}{\pi t}$$

$x_1(t)$ and $x_2(t)$ are two signals then

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

Here $x_1(t) = x(t)$

$x_2(t) = \frac{1}{\pi t}$

Instead of τ, here we have used $\underline{\underline{\alpha}}$.

* $\text{sgn}(t) = \frac{1}{j\pi f}$

so using duality property of Fourier transform.

$g(t) \Leftrightarrow G(f)$ then $G(t) \Leftrightarrow g(-f)$

so, incase of $\text{sgn}(t)$

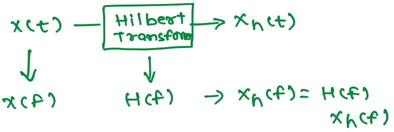
$\frac{1}{j\pi t} \Leftrightarrow \text{sgn}(-f) \Leftrightarrow$

$\frac{1}{\pi t} \Leftrightarrow -j\,\text{sgn}(f)$

Also using convolution property

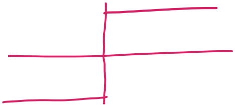
$x_1(t) * x_2(t) \Leftrightarrow X_1(f)\, X_2(f)$

$x(t) * \frac{1}{\pi t} \Leftrightarrow X(f)\,[-j\,\text{sgn}(f)]$



Transfer function.

$$H(f) = -j\,\mathrm{sgn}(f)$$



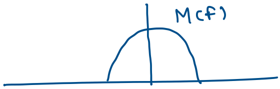
$\Rightarrow \mathrm{sgn}(f)$

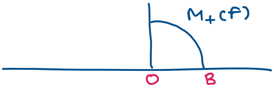
$$H(f) = -j = 1\cdot e^{-j\pi/2} \;;\; f>0$$

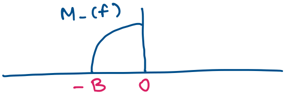
$$= j = 1\cdot e^{j\pi/2} \;;\; f<0$$

Hilbert Transform is ideal phase-shifter which shifts the spectrum of every component by $\underline{\underline{-\pi/2}}$.

*







$$M_+(f) = M(f)\,u(f)$$

$$u(f) = \frac{1}{2}\left[1 + \mathrm{sgn}(f)\right]$$

$$\Rightarrow M_+(f) = \frac{1}{2}M(f) + \frac{M(f)}{2}\,\mathrm{sgn}(f)$$

$$= \frac{1}{2}M(f) - \frac{j\,\mathrm{sgn}(f)}{-j}\,M(f)$$

$$M^+(f) = \frac{1}{2}M(f) + \frac{j\,M_h(f)}{2}$$

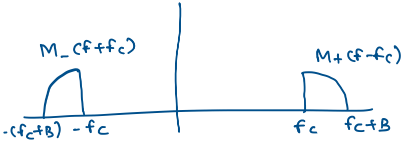
Similarly,

$$M^-(f) = M(f)\,u(-f)$$

$$= \frac{M(f)}{2}\left[1 - \operatorname{sgn}(f)\right]$$

$$\Rightarrow M^{-}(f) = \frac{1}{2}\left[M(f) - j M_h(f)\right]$$

$$\phi_{USB}(f) = M_{+}(f - f_c) + M_{-}(f + f_c)$$



$$\therefore\ \phi_{USB}(f) = \frac{1}{2}\left[M(f-f_c) + j M_h(f-f_c)\right]$$

$$+ \frac{1}{2}\left[M(f+f_c) - j M_h(f+f_c)\right]$$

$$\Rightarrow \phi_{USB}(f) = \frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

$$+ \frac{1}{2j}\left[M_h(f+f_c) - M_h(f-f_c)\right]$$

* Using Freq^n- shifting property of Fourier transform:

$$M(f - f_c) \Leftrightarrow m(t)\, e^{-j2\pi f_c t}$$

$$\phi_{USB}(t) = \frac{m(t)}{2}\left[e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right]$$

$$- m_h(t)\, \frac{\left[e^{j2\pi f_c t} - e^{-j2\pi f_c t}\right)}{2j}$$

$$= m(t)\cos(\omega_c t) - m_h(t)\sin(\omega_c t)$$

where, $m_h(t)$ → Hilbert Transform of signal $m(t)$

Similarly, it can be shown that

$$\phi_{LSB}(t) = m(t)\cos(\omega_c t) + m_h(t)\sin(\omega_c t)$$

It is possible to coherently demodulate this SSB signal.

$$\phi_{SSB}(t) \times 2\cos(\omega_c t)$$

$$= [m(t)\cos(\omega_c t) - m_h(t)\sin(\omega_c t)] \times 2\cos(\omega_c t)$$

$$= 2m(t)\cos^2(\omega_c t) - m_h(t)\sin(\omega_c t)\, 2\cos(\omega_c t)$$

$$= m(t)[1+\cos(2\omega_c t)] - m_h(t)\sin(2\omega_c t)]$$

$$= m(t) + m(t)\cos(2\omega_c t) - m_h(t)\sin(2\omega_c t)$$

$\downarrow$

can be filtered out using Low pass filter.

* How to generate SSB-SC signal ??

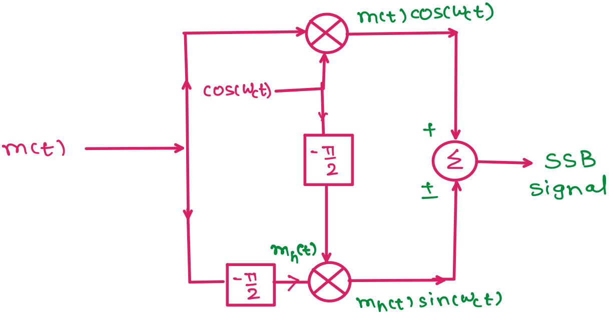
* Two Methods for SSB generation

1) Phase shift Method
2) Selective filter Method
3) Weaver's Method (Third method)

* Phase shift Method:

$$\phi_{USB}(t) = m(t)\cos(\omega_c t) - m_h(t)\sin(\omega_c t)$$

$$\phi_{LSB}(t) = m(t)\cos(\omega_c t) + m_h(t)\sin(\omega_c t)$$



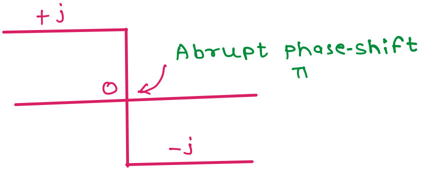
where $\otimes$ is balance modulator

$\boxed{-\frac{\pi}{2}}$ is Hilbert Transformer

Issues with phase-shift Method:

→ Ideal phase-shifter is unrealizable because Hilbert phase-shift requires abrupt phase change "π" at zero frequency.

$$H(f) = -j\,\mathrm{sgn}(f)$$

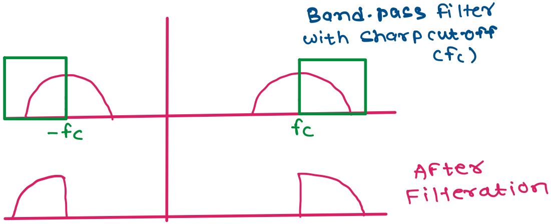


→ When signal has DC-null (no freq^n component at zero) and very little low freq^n content then practical approximation of Hilbert phase-shifter can be used for SSB generation.

→ But in-general, such phase-shifter circuits are very difficult to realize.

* Selective Filtering Method:

→ It is most common method for SSB generation.



→ To generate SSB signal, DSB-SC signal is passed through sharp-cut-off filter to eliminate un-wanted sideband.

→ But actual filters do not have sharp cut-off.

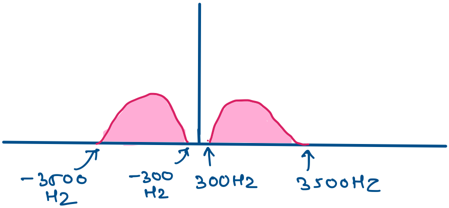
→ Actual filters do have some roll-off. And because of that it is not possible to fully suppress the other sideband.

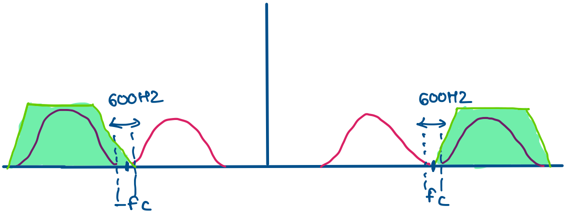
→ However, there are some signals which has no-freq^n components at origin. And low-freq^n components are insignificant.

→ voice signal is one such signal.

→ For voice signals, freq^n components below 300Hz are not important and can be suppressed.

→ In such case, selective filtering can be used.





→ As, shown above for voice signal, using band-pass filter, it is possible to suppress either upper or the lower sideband.

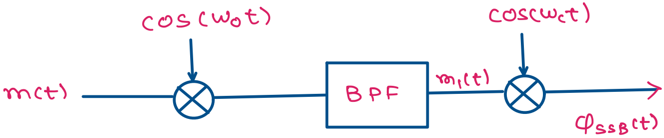
→ The undesired sideband should be attenated by atleast 40dB.

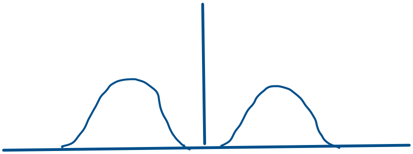
→ But when f_c is very high then the ratio of gap band to carrier freq^n is very small and it is difficult to achieve transistion of 40 dB in amplitude over 600Hz.

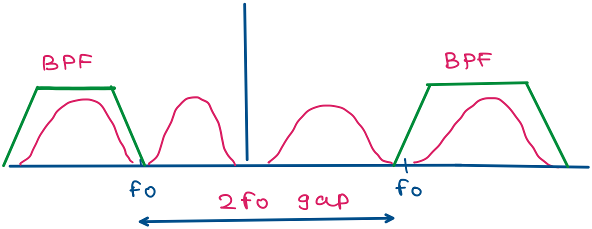
* In such case, the signal is first modulated to intermediate freq^n and using band-pass filtering the unwanted side-band is removed.

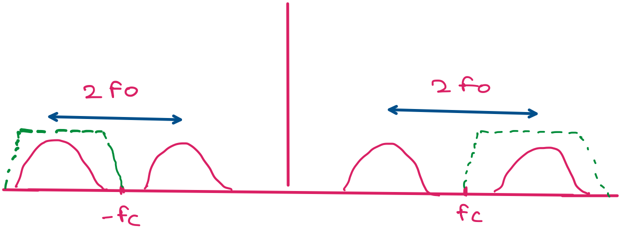
→ Because at lower freq^n, it is possible to achieve 40dB transition.

→ once the unwanted sideband is removed, the modulation is carried out at the required carrier freq^n.





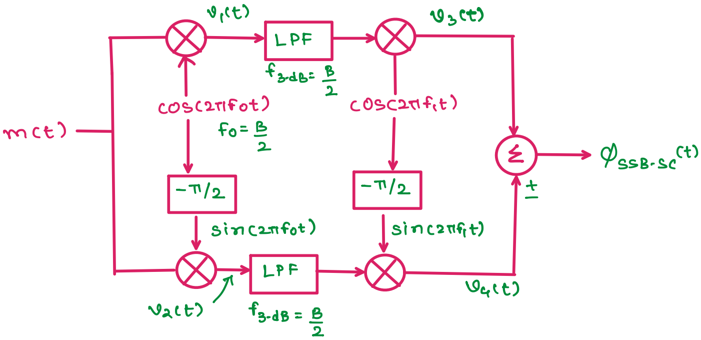




→ When modulation is carried out in two stages, the gap betbeen the two band widens ($\underline{2f_0}$).

→ And now it is possible to attenate the unwanted band by atleast 40dB using band pass filter.

3) Weaver's Method (Third Method)



If message signal m(t) has maximum freq^n component $\underline{B}$, then

→ cut-off freq^n of LPF = $\underline{\frac{B}{2}}$

→ carrier freq^n $\underline{f_0 = \frac{B}{2}}$

→ carrier freq^n $f_1 = f_c + \frac{B}{2}$ (For USB)

$= f_c - \frac{B}{2}$ (for LSB)

→ To obtain the upper-side band add $v_3(t)$ and $v_4(t)$

→ To obtain the Lower-side band perform $v_3(t) - v_4(t)$.

* Unlike phase-shift method, this weaver's method does not require wide band phase shifter circuit.

* It requires a phase-shifter (-90°) for only single freq^n. And hence complexity of the phase-shift circuit reduces substantially.

* Mathematical proof of weaver's Method (For single-tone modulation)

Let's say $m(t) = \cos(2\pi f_m t)$

Therefore,

$$v_1(t) = m(t)\cos(2\pi f_0 t)$$
$$= \cos(2\pi f_m t)\cos(2\pi f_0 t)$$
$$= \frac{1}{2}\left[\cos(2\pi(f_m+f_0)t) + \cos[2\pi(f_m-f_0)t]\right]$$

since, $f_0 = \frac{B}{2} = f_m/2$

$$v_1(t) = \frac{1}{2}\left[\cos(2\pi(1.5f_m)t) + \cos(2\pi(\frac{f_m}{2})t)\right]$$

The cut-off freq^n of LPF = $f_m/2$

Therefore, it will pass only one component.

Let's say, the signal after LPF is $v_1^*(t)$

$$\Rightarrow v_1^*(t) = \frac{1}{2}\cos\left[2\pi \times \frac{f_m}{2} \times t\right]$$

→ Now,

$$v_3(t) = v_1^*(t) \times \cos(2\pi f_1 t)$$

$$= \frac{1}{2}\cos\left[2\pi \frac{f_m}{2} t\right] \times \cos(2\pi f_1 t)$$

$$v_3(t) = \frac{1}{4}\left[\cos\left[2\pi\left(f_1 + \frac{f_m}{2}\right)t\right) + \cos\left(2\pi\left(f_1 - \frac{f_m}{2}\right)t\right)\right]$$

similarly,

$$v_2(t) = m(t)\sin(2\pi f_0 t)$$

$$= \cos(2\pi f_m t)\sin(2\pi f_0 t)$$

$$= \frac{1}{2}\left[\sin(2\pi(f_0 + f_m)t) + \sin(2\pi(f_0 - f_m)t)\right]$$

since, $f_0 = \frac{f_m}{2}$,

$$v_2(t) = \frac{1}{2}\left[\sin(2\pi \times 1.5 f_m t) - \sin\left(2\pi\left(\frac{f_m}{2}\right)t\right)\right]$$

After low pass filter, the freqⁿ component 1.5fm will get suppressed.

If $v_2^*(t)$ is the output of the low-pass filter,

$$v_2^*(t) = -\frac{1}{2}\sin\left(2\pi\left(\frac{f_m}{2}\right)t\right)$$

Now,

$$v_4(t) = v_2^*(t)\sin(2\pi f_1 t)$$

$$= -\frac{1}{2}\sin\left(2\pi\left(\frac{f_m}{2}\right)t\right)\sin(2\pi f_1 t)$$

$$v_4(t) = \frac{1}{4}\left[\cos\left(2\pi\left(f_1 + \frac{f_m}{2}\right)t\right) - \cos\left(2\pi\left(f_1 - \frac{f_m}{2}\right)\right)\right]$$

when $v_3(t) + v_4(t)$

output $\varphi_{SSB\text{-}SC}(t) = \frac{1}{2}\cos\left(2\pi\left(f_1 + \frac{f_m}{2}\right)t\right)$

And f_1 shoud be $f_c + f_m/2$

$$\therefore\ \phi_{SSB-SC}(t) = \underbrace{\frac{1}{2}\cos(2\pi(f_c + f_m)t)}_{\text{upper-sideband}}$$

Likewise, when

$$v_3(t) - v_4(t)$$

$$= \frac{1}{2}\cos\left(2\pi\left(f_1 - \frac{f_m}{2}\right)t\right)$$

and to obtain lower-sideband

$$f_1 = f_c - f_m/2$$

$$\Rightarrow\ \phi_{SSB-SC}(t) = \underbrace{\frac{1}{2}\cos(2\pi(f_c - f_m)t)}_{\text{lower-sideband}}$$

* SSB Tone modulation:

* SSB modulation:

$$\phi_{SSB}(t) = m(t)\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t)$$

'−' sign for USB

'+' sign for LSB.

Let's say, $m(t) = \cos\omega_m t$

$$\phi_{SSB}(t) = \cos(\omega_m t)\cos(\omega_c t) \mp \cos\left(\omega_m t - \frac{\pi}{2}\right)\sin(\omega_c t)$$

where, $m_h(t) = \cos\left(\omega_m t - \frac{\pi}{2}\right) = \sin(\omega_m t)$

Therefore,

$$\phi_{SSB}(t) = \cos(\omega_m t)\cos(\omega_c t) \mp \sin(\omega_m t)\sin(\omega_c t)$$

$$\therefore\ \phi_{SSB}(t) = \cos[(\omega_c \mp \omega_m)t]$$

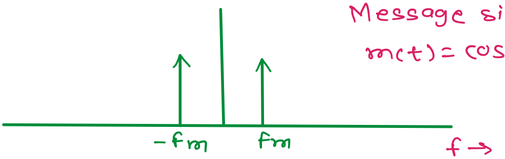
$$\Rightarrow \phi_{USB}(t) = \cos(\omega_c+\omega_m)t$$

$$\phi_{LSB}(t) = \cos(\omega_c-\omega_m)t$$

In freq^n domain:

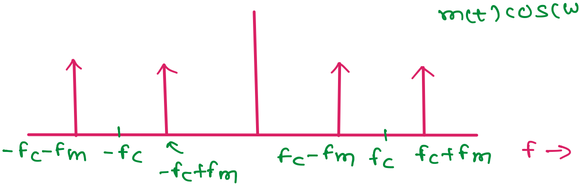
Message signal

$m(t) = \cos(\omega_m t)$

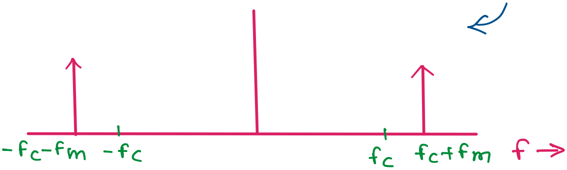


DSB-SC signal

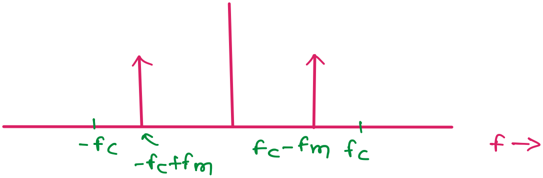
$m(t)\cos(\omega_c t)$



USB spectrum



LSB spectrum



* The SSB tone modulated signal can be demodulated coherently.

$$\phi_{SSB}(t)\cdot[2\cos(\omega_c t)] = 2\cos(\omega_c \pm \omega_m)t \times \cos(\omega_c t)$$

$$= \underline{\cos(\omega_m t)} + \cos(2\omega_c \pm \omega_m)t$$

can be passed
through LPF

* Power saving in SSB Modulation:

→ Since only one sideband is passed in SSB modulation, there is a power saving compared to DSB-SC or conventional AM.

→ For tone modulation,

If P_c is carrier power, then

power in two sidebands $= \frac{P_c \mu^2}{2}$

where $\mu \rightarrow$ modulation Index

In DSB-SC, power saving

$$= 1 - \frac{\text{Useful power}}{\text{Total power}}$$

$$= 1 - \frac{P_{SB}}{P_c + P_{SB}} = 1 - \frac{P_c \mu^2/2}{P_c + P_c \mu^2/2}$$

$$- \quad P_c \qquad = \quad \frac{2}{}$$

$$- \quad \overline{P_c\left(1+\frac{\mu^2}{2}\right)} \qquad 2+\mu^2$$

* For SSB tone modulation:

$$\text{Useful power} = \frac{P_{SB}}{2} = \frac{P_c\mu^2}{4}$$

$$\therefore \text{power saved} = \text{Total power} - \text{Useful power}$$

$$= \left[P_c + \frac{P_c\mu^2}{2}\right] - \frac{P_c\mu^2}{4}$$

$$= P_c + \frac{P_c\mu^2}{4}$$

$$\therefore \text{\% of power saved} = \frac{\text{power saved}}{\text{Total power}}$$

$$= \frac{P_c + P_c\mu^2/4}{P_c + P_c\mu^2/2}$$

$$= \frac{1 + \mu^2/4}{1 + \mu^2/2}$$